Consider the arithmetic sequence with $a_7 = 26$ and $a_{15} = 14$.

SCORE: ____ / 9 PTS

[a] Find the formula for a_n.

$a_7 = a_1 + 6d = 26$

$a_{15} = a_1 + 14d = 14$ ⊖ (1)

$8d = -12$

$d = -\frac{3}{2}$ (1)

$a_1 - 9 = 26$

$a_1 = 35$ (½)

$a_n = 35 - \frac{3}{2}(n-1) = \frac{73}{2} - \frac{3}{2}n$ (2)

EITHER VERSION OK

[b] -40 is a term in the sequence. Find the sum of all terms in the sequence up to and including that term.

$\frac{73}{2} - \frac{3}{2}n = -40$ (1)

$-\frac{3}{2}n = -\frac{153}{2}$

$n = 51$ (½)

$S_{51} = \frac{51}{2}(35 + -40)$ (2)

$= \frac{-255}{2}$ (1)

Write $\frac{25}{96}-\frac{36}{48}+\frac{49}{24}-\frac{64}{12}+\frac{81}{6}-\frac{100}{3}$ using sigma notation.

NUMERATORS: PERFECT SQUARES

DENOMINATORS: GEOMETRIC $r=\frac{1}{2}$

SCORE: _____ / 4 PTS

$$\sum_{n=1}^{6}(-1)^{n+1}\frac{(n+4)^2}{96\cdot\left(\frac{1}{2}\right)^{n-1}}=\sum_{n=1}^{6}(-1)^{n+1}\frac{2^{n-1}(n+4)^2}{96}$$

(-1) POINT IF YOU USED A DIFFERENT INDEX (LETTER) UNDER THE $\sum$ VERSUS IN THE FORMULA

Find the 4^{th} term of the sequence defined recursively by $a_1 = -1, \quad a_2 = 2, \quad a_n = na_{n-2} - 2a_{n-1}$ for $n \geq 3$. **SCORE:** _____ / 4 PTS

$a_3 = 3a_1 - 2a_2 = 3(-1) - 2(2) = -7$

$a_4 = 4a_2 - 2a_3 = 4(2) - 2(-7) = 22$

(1) ~~EACH~~

Find the value of $\sum_{i=2}^{5}(-1)^{i+1}(i!-i^3)$.

SCORE: _____ / 5 PTS

$= -(2-8)+(6-27)-(24-64)+(120-125)$

$= 6-21+40-5$

$= 20$

(1) EACH

Simplify $\frac{(2k-6)!}{(2k-4)!}$.

SCORE: _____ / 3 PTS

$$\frac{(2k-6)!}{(2k-4)(2k-5)(2k-6)!} = \frac{1}{(2k-4)(2k-5)}$$

(2) (1)

Find the sum of the first 11 terms of the sequence $\frac{15}{8}-\frac{3}{2}+\frac{6}{5}-\frac{24}{25}+\cdots$.

SCORE: ____ / 5 PTS

GEOMETRIC: $r=\frac{-\frac{3}{2}}{\frac{15}{8}}=-\frac{3}{2}\cdot\frac{8}{15_5}=\boxed{-\frac{4}{5}}$ (1)

CHECK: $-\frac{3}{2}\cdot-\frac{4}{5}=\frac{6}{5}$, $\frac{6}{5}\cdot-\frac{4}{5}=-\frac{24}{25}$

(3)

$$S_{11}=\boxed{\frac{\frac{15}{8}\left(1-\left(-\frac{4}{5}\right)^{11}\right)}{1-\left(-\frac{4}{5}\right)}}=\frac{\frac{15}{8}\left(1+\left(\frac{4}{5}\right)^{11}\right)}{\frac{9}{5}}=\frac{5}{9_3}\cdot\frac{15^5}{8}\left(1+\left(\frac{4}{5}\right)^{11}\right)=\frac{25}{24}\left(1+\left(\frac{4}{5}\right)^{11}\right)\approx 1.1311$$

EITHER VERSION OK (1)